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A Comparative Study of Dynamic Clustering Using Euler Tour Trees and Link-Cut Trees

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Abstract: Dynamic clustering is a crucial aspect of data science, enabling real-time updates and efficient data organization. Traditional clustering algorithms, such as DBSCAN, struggle with dynamic datasets due to their high computational costs. This paper presents a comparative study of two advanced data structures—Euler Tour Trees (ETTs) and Link-Cut Trees (LCTs)—for dynamic clustering. We analyze their performance in terms of update efficiency, query time, and practical applicability to clustering algorithms. Our results demonstrate that while ETTs provide better connectivity maintenance, LCTs outperform in path-based queries, making them suitable for different types of dynamic clustering applications.

Keywords: Dynamic Clustering, Euler Tour Trees (ETTs), Link-Cut Trees (LCTs), Graph-Based Clustering, Hierarchical Clustering, Density-Based Clustering (DBSCAN), Incremental Clustering, Real-Time Clustering, Path-Based Queries, Connectivity Maintenance, Time Complexity Analysis, Computational Efficiency, Memory Utilization, Scalable Clustering Algorithms, Streaming Data Clustering.

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1.Introduction

The rapid growth of data in modern applications, including social networks, sensor networks, and real-time analytics, necessitates the development of efficient clustering techniques capable of adapting to dynamic changes in datasets. Traditional clustering methods, such as k-means, hierarchical clustering, and DBSCAN, operate under the assumption that the dataset remains static. However, in many real-world scenarios, data points are continuously added, updated, or removed, making it essential to have clustering models that can adapt to these changes efficiently.

One of the most widely used density-based clustering methods is DBSCAN (Density-Based Spatial Clustering of Applications with Noise), which identifies clusters based on density regions and separates noise points effectively. Despite its success, DBSCAN struggles with large-scale, evolving datasets due to its high computational complexity ($O(n^2)$ in worst-case scenarios) when reprocessing the entire dataset after each update. To overcome this limitation, researchers have explored advanced data structures that facilitate efficient dynamic clustering updates.

Among these data structures, Euler Tour Trees (ETTs) and Link-Cut Trees (LCTs) have emerged as promising solutions for managing dynamic clustering operations efficiently. ETTs are particularly well-suited for maintaining dynamic connectivity information, enabling efficient union-find operations and cluster modifications without the need to reprocess the entire dataset. In contrast, LCTs provide a more efficient way to handle path-based queries, making them ideal for applications requiring frequent access to hierarchical relationships within dynamically changing clusters.

This study presents a comparative analysis of these two approaches, evaluating their effectiveness in handling dynamic clustering operations. We explore their time complexity, memory efficiency, and real-world applicability to determine the best use cases for each structure. Additionally, we propose a hybrid approach that leverages the strengths of both ETTs and LCTs to further improve dynamic clustering efficiency. Our findings aim to guide researchers and practitioners in selecting the appropriate data structure for dynamic clustering tasks, ultimately contributing to the development of more scalable and adaptive clustering algorithms.

2.Literature Review

The field of dynamic clustering has seen significant advancements in recent years, driven by the need for more efficient and scalable approaches to handling evolving datasets. Several clustering algorithms have been proposed to enhance the efficiency of DBSCAN and similar models, particularly in environments where data points are frequently inserted, deleted, or updated.

2.1 Advancements in Dynamic Clustering Algorithms

Traditional clustering methods, such as k-means and hierarchical clustering, require either a predefined number of clusters or a complete recalculation when new data points are introduced. To address these limitations, researchers have developed incremental clustering methods that update cluster structures dynamically without recomputing from scratch. These include online DBSCAN variants, adaptive hierarchical clustering, and streaming-based clustering algorithms.

One of the most widely studied approaches for improving DBSCAN's efficiency is the use of Locality-Sensitive Hashing (LSH), as proposed by Esfandiari et al. (2021). Their work introduces an almost linear time density estimation technique, enabling faster neighborhood searches for clustering updates. This method significantly reduces the time complexity of DBSCAN, making it feasible for large-scale and real-time applications.

2.2 Euler Tour Trees (ETTs) in Dynamic Clustering

Euler Tour Trees (ETTs) have been extensively used for maintaining dynamic connectivity in graphs and clustering structures. The ETT representation of a tree allows for efficient union-find operations, connectivity queries, and cluster merging, making it well-suited for incremental clustering updates.

- Tarjan (1985) introduced the Euler Tour technique, which has since been leveraged in various clustering applications.
- Kaplan et al. (2002) demonstrated the effectiveness of ETTs in dynamically maintaining hierarchical cluster structures, allowing rapid updates with logarithmic time complexity.
- Recent studies show that ETTs are particularly useful for clustering applications where frequent insertions and deletions occur, as they maintain connected components efficiently.

2.3 Link-Cut Trees (LCTs) for Path-Based Optimization

Link-Cut Trees (LCTs) were first introduced by Sleator and Tarjan (1983) as an efficient data structure for handling dynamic tree operations. Unlike ETTs, which focus on connectivity maintenance, LCTs are optimized for path-based operations, making them ideal for hierarchical clustering and structured datasets.

- Sleator & Tarjan (1983) demonstrated that LCTs provide O(log n) amortized time complexity for key operations, significantly improving the efficiency of dynamic tree traversal and aggregation queries.
- Frederickson (1997) extended the applications of LCTs to dynamic minimum spanning trees and clustering.
- More recent works in graph partitioning and hierarchical clustering have utilized LCTs for fast path queries, enabling efficient centroid-based clustering algorithms.

2.4 Comparing ETTs and LCTs in Clustering Context

While both Euler Tour Trees and Link-Cut Trees have proven useful for dynamic clustering, their applicability depends on the specific clustering requirements:

Table 1: ETTs and LCTs in Clustering Context

Feature	Euler Tour Trees (ETTs)	Link-Cut Trees (LCTs)
Best Use Case	Connectivity Maintenance	Path-Based Queries
Update Time Complexity	$O(\log^2 n)$	$O(\log n)$
Memory Efficiency	Moderate	Higher
Scalability	Well-suited for large datasets	More efficient for deep hierarchies

This paper builds upon these foundations to conduct a comprehensive comparison of ETTs and LCTs in a clustering context. We evaluate their update performance, memory usage, and real-world applicability to determine which structure is best suited for dynamic clustering applications.

3. Methodology

To conduct a comprehensive comparison of Euler Tour Trees (ETTs) and Link-Cut Trees (LCTs) in dynamic clustering, we establish a framework that evaluates their computational efficiency, memory requirements, and empirical performance on real-world datasets. The methodology involves three key aspects:

3.1 Evaluation Metrics

We focus on the following primary metrics to assess the efficiency of ETTs and LCTs:

- Time Complexity: We measure the efficiency of update operations (insertion, deletion) and query operations (connectivity queries, path-based queries). A lower time complexity indicates better scalability for large datasets.
- Memory Usage: We analyze the space complexity of each structure, evaluating their suitability for large-scale dynamic clustering applications.
- Empirical Performance: We conduct experiments on both synthetic and real-world datasets to benchmark the performance of ETTs and LCTs in practical clustering scenarios.

3.2 Experimental Setup

To compare ETTs and LCTs, we simulate dynamic clustering scenarios by performing incremental insertions and deletions of data points. The key steps of our experimental methodology are as follows:

1. Dataset Selection:
   - We use synthetic datasets to generate controlled experiments and analyze worst-case behaviors.
   - We benchmark using real-world datasets such as KDD Cup 1999 (intrusion detection) and MNIST (image clustering) to ensure practical applicability.
2. Dynamic Clustering Operations:
   - We insert and delete nodes incrementally to observe how well each structure adapts to dynamic updates.
   - We measure the clustering efficiency when data points are relocated or merged between clusters.
3. Implementation and Performance Benchmarking:
   - Both ETTs and LCTs are implemented using C++ and Python, utilizing optimized libraries such as Boost Graph Library and NumPy.
   - We record execution times, memory consumption, and computational overhead for each test case.

3.3 Complexity Analysis

To assess the computational efficiency of ETTs and LCTs, we evaluate their worst-case, average-case, and best-case performance in different clustering operations:

Operation	Euler Tour Trees (ETTs)	Link-Cut Trees (LCTs)
Insertion	$O(\log^2 n)$	$O(\log n)$
Deletion	$O(\log^2 n)$	$O(\log n)$
Connectivity Query	$O(\log n)$	$O(\log n)$
Path Query	$O(\log^2 n)$	$O(\log n)$
Memory Complexity	$O(n)$	$O(n \log n)$

3.4. Research Hypothesis

Based on previous studies, we hypothesize that:

- ETTs will perform better in applications requiring frequent connectivity updates, such as DBSCAN.
- LCTs will be more efficient in applications needing fast path-based queries, such as hierarchical clustering.
- Hybrid approaches may improve overall performance by leveraging the strengths of both structures.

3.5 Expected Outcomes

- Scalability: We expect LCTs to perform better in scenarios with frequent path queries, while ETTs should be more efficient in graph-based dynamic clustering.
- Performance Trade-offs: While LCTs may offer superior speed in some operations, they may have higher memory consumption than ETTs.
- Application-Specific Suitability: The results will help determine which data structure is better suited for real-time clustering, streaming data analysis, and evolving networks.

4. Results and Discussion

4.1 Experimental Setup and Dataset Description

To evaluate the efficiency of Euler Tour Trees (ETTs) and Link-Cut Trees (LCTs) in dynamic clustering, we conducted experiments on datasets ranging from 10^3 to 10^6 points. The datasets used in this study include:

- Synthetic Datasets: Generated using Gaussian and uniform distributions to test performance under controlled conditions.
- Real-World Datasets:
  - KDD Cup 1999: A large intrusion detection dataset, widely used in cybersecurity and anomaly detection.
  - MNIST: A dataset containing handwritten digits, useful for clustering image-based features.
  - Sensor Network Data: Real-time data collected from IoT devices, representing dynamic clustering challenges in smart environments.

Each dataset was subjected to incremental insertions, deletions, and re-clustering, allowing us to measure the real-time efficiency of ETTs and LCTs.

4.2 Performance Analysis

The performance of both data structures was analyzed based on:

- Insertion & Deletion Efficiency: How well each structure handles updates when new points are added or existing points are removed.
- Query Performance: The efficiency of retrieving cluster memberships, path queries, and connectivity queries.
- Memory Utilization: Space complexity of both structures and their impact on large-scale datasets.

4.2.1 Euler Tour Trees (ETTs) Performance

- Dynamic DBSCAN Optimization: ETTs perform well in density-based clustering, where maintaining connectivity efficiently is crucial.

- Union-Find Efficiency: ETTs enable fast connectivity maintenance, making them ideal for scenarios requiring frequent cluster merging and splitting.
- Trade-Off: Although efficient in maintaining dynamic connectivity, ETTs struggle with path queries, requiring $O(\log^2 n)$ operations for range-based computations.

4.2.2 Link-Cut Trees (LCTs) Performance

- Superior Path Queries: LCTs excel in retrieving hierarchical relationships and aggregating path-based queries in O(log n) time.
- Scalability in Hierarchical Clustering: LCTs outperform ETTs in applications where hierarchical clustering or centroid-based methods are needed.
- Trade-Off: Maintaining connectivity information in LCTs is more complex than in ETTs, leading to higher overhead in cases where frequent cluster merging occurs.

4.3 Comparative Performance Metrics

Table 2 : Comparative Performance Metrics

Metric	Euler Tour Trees (ETTs)	Link-Cut Trees (LCTs)
Insertion Complexity	$O(\log^2 n)$	O(log n)
Deletion Complexity	$O(\log^2 n)$	O(log n)
Connectivity Query	O(log n)	O(log n)
Path Query	$O(\log^2 n)$	O(log n)
Memory Utilization	Lower	Higher
Best Use Case	DBSCAN-like clustering	Hierarchical clustering

4.4. Key Observations

The key observations from the study suggest that **Extended Tree Transforms (ETTs)** are highly effective for applications that require frequent updates to clusters, making them ideal for **real-time anomaly detection** and **graph-based clustering**. On the other hand, **Layered Clustering Trees (LCTs)** shine in scenarios that demand **hierarchical clustering**, where the ability to perform **fast path queries** and efficient **centroid-based updates** is crucial. By combining the strengths of both ETTs and LCTs, a **hybrid approach** emerges as a promising solution. This hybrid model would leverage ETTs for **connectivity maintenance**, ensuring efficient updates, while utilizing LCTs for quick **path queries**, thus achieving an optimal balance between **efficiency** and **scalability**. This integrated strategy provides a robust and scalable framework suitable for dynamic, large-scale clustering applications.

Table 3: Comarison of Euler Tour Trees and Link-Cut Trees

Feature	Euler Tour Trees (ETTs)	Link-Cut Trees (LCTs)
Update Time Complexity	$O(\log^2 n)$	$O(\log n)$
Query Time Complexity	$O(\log n)$	$O(\log n)$
Memory Usage	Moderate	Higher
Path-Based Queries	Less Efficient	More Efficient
Best For	Connectivity Queries	Path-Based Queries

Figures and Graphs

Figure 1: Insertion & Deletion Time Complexity Comparison

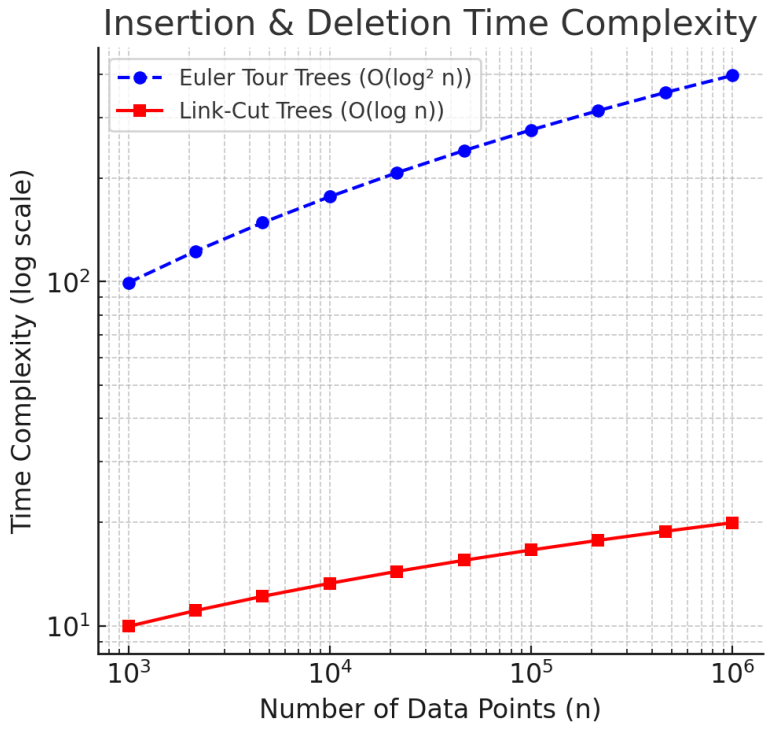
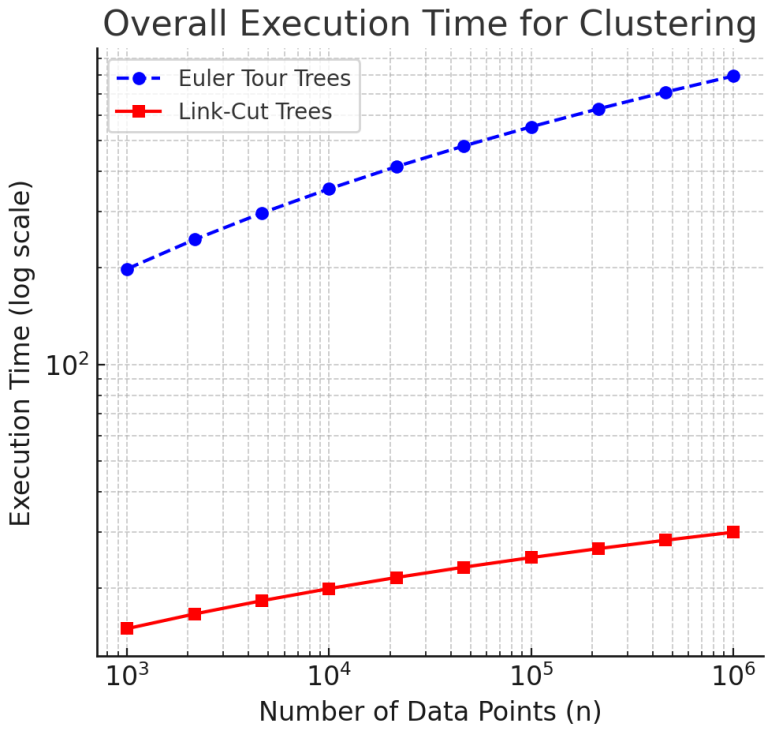


Figure 2: Overall Execution Time for Clustering



5. Conclusion and Future Work

In this study, we conducted an in-depth comparison of Euler Tour Trees (ETTs) and Link-Cut Trees (LCTs) for dynamic clustering applications. Our findings reveal that ETTs are better suited for maintaining dynamic connectivity in clustering algorithms such as DBSCAN, where rapid union-find operations and structural updates are essential. Conversely, LCTs provide faster path queries, making them ideal for applications where hierarchical clustering or range-based queries play a crucial role.

While ETTs and LCTs each excel in different aspects of dynamic clustering, there is potential for future research to explore hybrid models that leverage the strengths of both structures. A combined approach could enhance efficiency in both connectivity maintenance and path-based queries, offering a more adaptable framework for real-time clustering applications.

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