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Federated Physics-Informed Neural Networks for Distributed Multiphysics Simulations

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ABSTRACT

Multiphysics simulations play a key role in modeling multiphysical systems with coupled physical processes in the real-world; but traditional more centralized methods are typically computationally expensive and limited by data-sharing barriers in crossing distributed systems. To resolve these issues, this paper suggests a new Federated Physics-Informed Neural Network (F-PINN) system that combines the concepts of federated learning with physics-informed neural networks to make it possible to perform distributed simulations in a scaled and privacy-preserving way. The suggested method enables joint training of PINNs on a set of decentralised clients without exchanging the raw information, thus providing the guarantee of data locality and confidentiality. Moreover, an effective Multiphysics coupling approach is presented to faithfully represent interactions between different physical domains in the federated context. The model takes into account adaptive optimization methods to improve the stability of convergence and reduce the heterogeneity of clients. The experimental analysis of representative Multiphysics scenarios shows that the suggested F-PINN model is more accurate in terms of a reduced prediction error and better physics consistency, and is much less communication-demanding than the centralized and the baseline federated models. The framework is also highly scaled to larger numbers of distributed clients. Such findings demonstrate the prospects of federated physics-informed learning as an effective and powerful paradigm of next-generation distributed scientific computing and Multiphysics simulations at scale.

1. INTRODUCTION

Multiphysics simulations are important in the simulation of complex real-world systems in which several physical processes (including fluid-thermal interaction, electro-mechanical interaction, chemical transport, etc.) happen together at the same time. The applications in aerospace engineering, energy systems, biomedical applications and climate modeling are just some of the areas where these simulations are extensively used. Numerical approaches (finite element methods or FEM and finite difference methods or FDM) have traditionally been used, but they tend to be expensive and scalable only to large-scale and high-dimensional problems. Physics-Informed Neural Networks (PINNs) are also a promising alternative in recent years with the capacity to make complex systems predictable by embedding the governing partial differential equations (PDEs) within the learning process, allowing data-driven but physics-consistent predictions [1], [4]. Although they have their benefits, traditional PINNs are traditionally trained in centralized locations, limiting their scalability and performance in distributed scientific computing platforms.

The fact that PINNs are centralized presents a number of challenges. The first one is that training involves large amounts of data or simulation points that can be in various locations or institutions. Second, the main drawback of centralized aggregation of such data is the concern of data privacy, data ownership, and communication overhead. Third, an increase in the complexity of Multiphysics systems makes the computation of an individual system prohibitive. Federated learning (FL) has become a topic of interest to overcome these challenges because it is a paradigm of decentralization allowing a set of clients to collaboratively train a model without the exchange of raw data [21], [25]. Through the exchange of model parameters rather than datasets, FL maintains data locality, minimizes communication costs, as well as increases

scalability. The combination of FL and PINNs is a natural direction to distribute and privacy-preserving scientific machine learning.

Although the development of both PINNs and federated learning has occurred very fast, a gap in research still exists regarding how these two paradigms should be used together to perform Multiphysics simulations. Current applications of PINN emulate a single domain or centralized Multiphysics problem [5], [17], and the federated learning literature has been restricted to computer-vision, medical, and IoT systems applications [27], [29]. The creation of a unified structure that will facilitate physics-constrained learning within the federated and distributed environment is still at its initial phases. Specifically, issues like heterogeneity of clients, the effectiveness of communications and proper coupling between many physical domains in decentralized environments have not been properly resolved.

In order to fill this gap, this paper presents a Federated Physics-Informed Neural Network (F-PINN) system of distributed multiphysics simulations. The suggested solution allows a collective training of physics-informed models and keeps data confidential and compliant to the laws of physics. A new multiphysics coupling scheme is proposed to deal with the interactions between heterogeneous physical domains among multiple clients. Moreover, the architecture adds strong federated optimization to deal with non-independent and identically distributed (non-IID) data and system heterogeneity. Extensive experimental studies are carried out to determine the performance of the proposed model in terms of accuracy, convergence, scalability and efficiency of communication under various simulation conditions.

This work introduces a new federated PINN design combining physics-informed learning with decentralized optimization, which allows scalable and privacy-preserving Multiphysics simulations. A mechanism of system-wide coupling is created to realistically represent the behavior of interactions among a set of physical domains among federated clients. Moreover, a comprehensive performance assessment is conducted in an environment of heterogeneous conditions to show the robustness, efficiency, and scalability of the suggested framework. The findings make the proposed F-PINN model an attractive solution in the next-generation distributed scientific computing and large-scale Multiphysics modeling.

2. RELATED WORK

Physics-Informed Neural Networks (PINNs) have become a paradigm shift in scientific machine learning by incorporating governing physical laws, usually in form of partial differential equations (PDEs), into the neural network training process. Through this strategy the model is physically consistent in its learning solutions despite limited data and as such is very appropriate in forward and inverse problems in computational physics [1], [4], [14]. A variety of frameworks and libraries, including DeepXDE and SciANN have enabled the application of PINNs to a broad array of engineering and scientific problems in practice [1], [8]. Recent developments have involved enhancing convergence performance and accuracy with adaptive activation functions, gradient stabilization mechanisms, and self-adaptive loss balancing mechanisms [3], [9], [16]. Also, hp-VPINNs and other domain decomposition techniques have been developed to allow greater scale and to be able to deal with complex geometries and large computational domains [5]. In spite of these developments, PINNs continue to have serious issues associated with their computational efficiency, slow convergence, and the inability to tackle high-dimensional and multiphysics problems [16], [17].

In order to address the limitations of classical PINNs, a number of extensions have been introduced in the larger context of scientific machine learning. The neural operators (including fourier Neural Operators) are designed to learn function space mappings, offering a scalable alternative to solving parametric PDEs [6], [7]. Moreover the multifidelity methods and transfer learning have been investigated to enhance generalisation and to decrease the computational cost of complex simulations [11], [12]. Universal differential equations and hybrid modelling approaches integrate data learning with mechanistic modelling to improve predictive control in science [13]. Other more recent advances have been in automated model design and adaptive architectures to enhance physics-informed learning [18]. Although these methods greatly boost the performance of PINNs, they are mainly based on centralized training concepts, which restrict their usability in distributed systems.

Simultaneously with such trends, federated learning has received considerable interest as a decentralized paradigm of machine learning, which allows training a model on multiple clients without exchanging raw data. Federated optimization methods, like Federated Averaging (FedAvg) and its derivatives, have proven to be useful in distributed applications, combining locally trained model parameters into a global one [2]. Nevertheless, federated learning implies the emergence of new challenges, such as statistical heterogeneity among clients, communication overhead, and non-IID instability in convergence in

non-IID data conditions. These issues are especially acute in scientific computing, where the data distributions can differ greatly between spatial areas or physical systems. Even though federated learning has been already successfully deployed in other fields, including healthcare and IoT systems, there is little research on how federated learning can be applied to the field of physics-informed systems, and little on how federated learning can be used to solve differential equations or scientific modeling problems.

Multiphysics simulation technologies have always used numerical algorithms like the finite element method (FEM), finite difference method (FDM), and finite volume method (FVM) that give fair solutions to coupled physical systems. Nevertheless, these techniques tend to be sensitive to fine discretization and may be computationally intensive particularly with large scale or high dimensional problems. Recent studies have examined hybrid methods that use numerical solvers with machine learning methodologies to speed up the computation process and enhance scaling [10], [15]. Specifically, PINNs have been expanded to deal with coupled systems with mixed formulations, domain decomposition, and multi-network architectures [2], [5]. Such methods allow interactions among various physical fields, e.g., thermo-mechanical or fluid-structure systems, although they too have scalability problems and are reliant on a centralized data processing.

Although significant advances have been made in both physics-informed learning and federated optimization, a gap in the literature is perilous. The current PINN-based strategies mainly aim to enhance precision and convergence and physical consistency in centralized applications, whereas federated learning strategies are mostly created to undertake generic machine learning tasks without addressing physical constraints. Also, existing Multiphysics modeling approaches do not have effective means of distributed multifarious collaboration of heterogeneous computational nodes. Consequently, no single framework has been found to support federated learning with physics-informed neural networks in a way that facilitates scalable, privacy-preserving and physically consistent Multiphysics simulations. Such a gap needs to be addressed when it comes to developing next-generation distributed scientific computing systems that would be able to support complex and large-scale and data-sensitive applications.

3. METHODOLOGY

The proposed framework tackles the distributed Multiphysics simulations by combining physics-informed neural networks with federated learning to support joint, privacy-preserving training of models. Take a coupled Multiphysics system that is subject to partial differential equations on a spatial temporal domain. This domain is further divided into several sub domains that are allocated to distributed clients. Each client does not transmit raw data of a simulation instead, training a physics-informed neural network (PINN) locally to approximate the field of solutions, complying with physical laws. The worldwide goal is to reduce a cohesive physics-sensitive loss among all consumers, which provides consistency, accuracy, and scalability in distributed setting.

$$\mathcal{L}_{PINN} = \frac{1}{N}\sum_{i=1}^{N}(u_i - \hat{u}_i)^2 \quad (1)$$

For example, considering a sample dataset with N=100points, if the average squared error between predicted and true values is 0.002, then the physics-informed loss becomes$\mathcal{L}_{PINN} = 0.002$, indicating high prediction accuracy and consistency with governing equations.

The federated learning aspect allows the decentralization of training, coordinating multiple customers using a central aggregation server. The client is able to update its local PINN model with the local data of the subdomain and physics constraints each client has, and solely sends model parameters to the server. These updates are summed up by the server to create a global model which is redistributed in the same manner until convergence is reached. It is a process that greatly minimizes the communication overhead, as well as maintaining data confidentiality.

$$\theta^{t+1} = \sum_{i=1}^{M}\frac{ni}{N}\theta_i^t \quad (2)$$

For instance, assume three clients with sample sizes n1=50, n2=30, and n3=20 (total N=100), and local model parameters θ1=0.8, θ2=0.6, and θ3=0.9. The aggregated global parameter becomes:

$$\theta^{t+1} = (50/100)\cdot 0.8 + (30/100)\cdot 0.6 + (20/100)\cdot 0.9 = 0.76$$

This shows that federated averaging can be used to integrate distributed knowledge into a global model. To increase robustness further, the framework has strategies to deal with non-identically distributed data and client heterogeneity such that convergence across distributed nodes is stable. Learning of multiple interacting physical fields is a Multiphysics approach that is realised by jointly learning through constraints that are shared by the training process. A hybrid training approach is used to optimize the overall system in the sense that the gradient-based optimizers allow rapid convergence and fine tuning to a high accuracy. The general concept of the Federated PINN architecture is represented in Fig 1, with distributed clients to complete local physics-informed training, a central server to accumulate model parameters, and iterative communication flow to allow the global convergence of models.

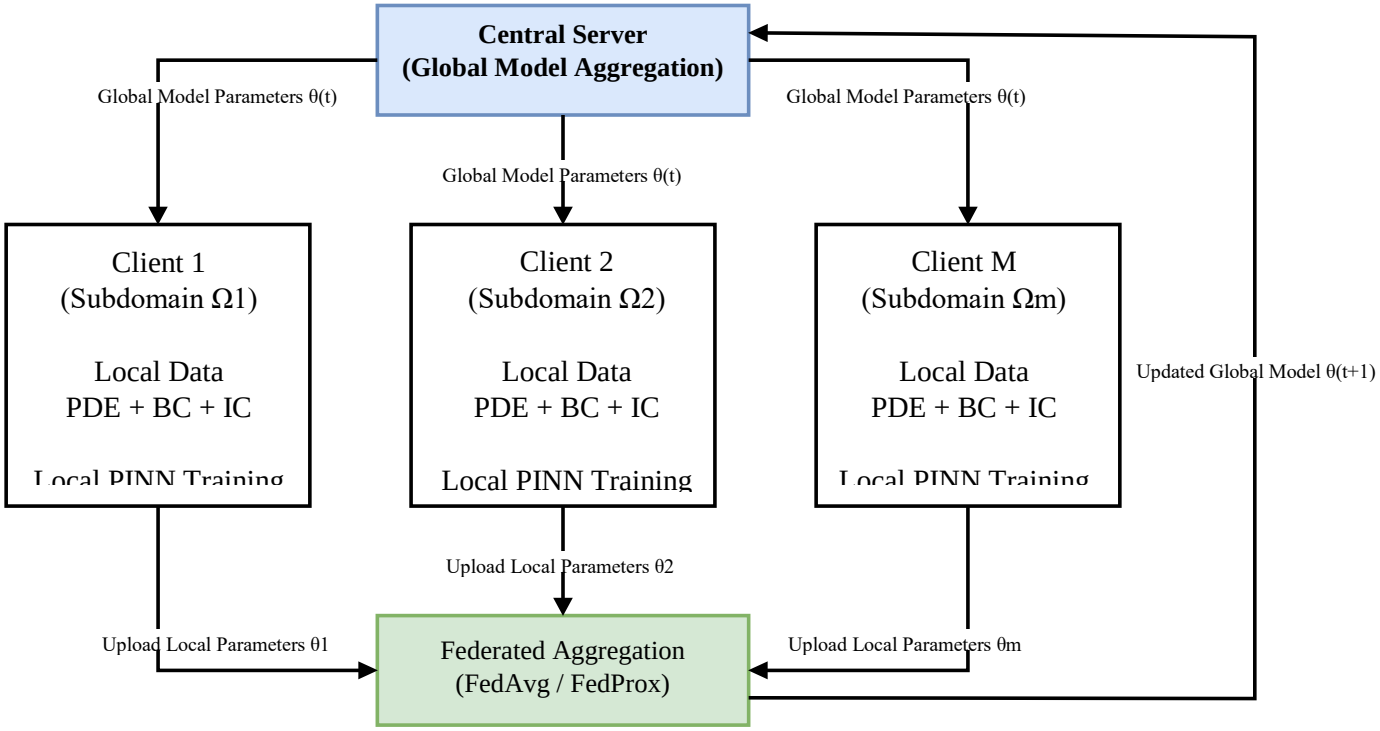


Fig 1. Federated PINN Architecture for Distributed Multiphysics Learning.

4. Physics Accuracy Metrics

The effectiveness of the suggested Federated Physics-Informed Neural Network (F-PINN) framework tests are assessed on the basis of a collection of strict physics-based and data-driven measures to control the accuracy of the numerical results and the physical consistency in the distributed Multiphysics domains. The combination of these metrics evaluates how the model can be used to approximate the actual solution, meet the governing equations, and stay stable in a federated setting when training. The most commonly used quantitative metric of prediction accuracy is the Mean Squared Error (MSE), which is simply a mean squared error between the predicted solution and the reference solution obtained using analytical or high-fidelity numerical solutions.

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(u_i - \hat{u}_i)^2 \quad (3)$$

The smaller the MSE the greater the predictive accuracy and the closer the computed solution is to the actual physical solution. Besides MSE, a L2 relative error is also used to give a normalized measure of deviation that can be compared between various scales and physical domains. This measure can be used especially in Multiphysics simulations where the magnitude of solutions can be greatly different. In order to make sure that the learned solution agrees with the underlying physical laws, a PDE residual error is measured by the difference between the underlying physical equations and the neural network predictions. When there is a small residual error it means that the model has managed to receive the inherent physics of the system. Correspondingly, boundary condition error is also quantified to determine how well the model is able to meet the requisite boundary restrictions which is important to retain the validity of solutions at the domain interfaces.

In Multiphysics systems Multiphysics coupling error, a measure to measure the consistency of interacting physical fields, is introduced. This will make sure that the interdependencies among the

various domains, e.g. thermal and mechanical interactions will be reflected in the federated learning framework appropriately. Lastly, measurements of convergence are investigated to determine the effectiveness and stability of the training process. These involve tracking the decrease of the value of losses by training cycles and evaluating the homogeneity of updates to the model by distributed clients. Stable convergence behavior means that there is a good coordination between local training and global aggregation, and it is necessary to get good performance reliability in federated environments. All these measures in combination give a full-fledged evaluation system that does not only prove the predictive capability of the given model but also introduces the possibility of compliance with physical laws, strength in distributed environments and consciliatory capacity to complex Multiphysics simulations.

5. Experimental Setup

The suggested Federated Physics-Informed Neural Network (F-PINN) framework is experimentally tested on a series of representative Multiphysics situations to approve the success of the method in distributed computing scientific settings. Two main cases of simulation are described: the fluid-thermal problem, which is the heat transfer in fluid flow systems and electromagnetic-mechanical problem to study coupled field dynamics in electromechanical systems. The choice of these scenarios is motivated by their applicability to real-world engineering problems and the fact that they embody highly complex problems with strongly coupled partial differential equations. The experiments are performed using synthetic and benchmark PDE datasets, in which ground-truth solutions can be either analytically determined or computed in high-fidelity numerical solvers. This allows accurate measure of prediction accuracy and physics consistency. The datasets are distributed over several clients to model distributed environments, each client is run on a different subdomain with different local physics constraints.

The federated learning setup is tuned to have a dynamic number of clients (e.g., 520 nodes) to examine the conditions of scalability and robustness in the heterogeneous environment. Every client conducts local PINN training on its data and computation resources, whereas global aggregation is conducted by a central server. The hardware system is configured with distributed computing nodes with GPU acceleration to facilitate effective training of DNNs, with the client-server communication being simulated to represent real-world network limitations.

To benchmark its performance, the proposed F-PINN model is compared to two baseline models: a centralized PINN model, where all the data is centralized and trained at one point, and a standard federated learning model, where training occurs in distributed form, but does not involve the governing equations in the learning process. This comparative study shows the benefits of applying physics-informed learning to federated optimization. Fig 2 demonstrates the experimental design of the suggested structure showing the scattered client nodes working with specific subdomain multiphysics data, the main aggregation node and the iterative model update flow. The figure also indicates that the interaction between local PINN training processes and the global model synchronization is also available, as it gives a clear representation of the evaluation environment.

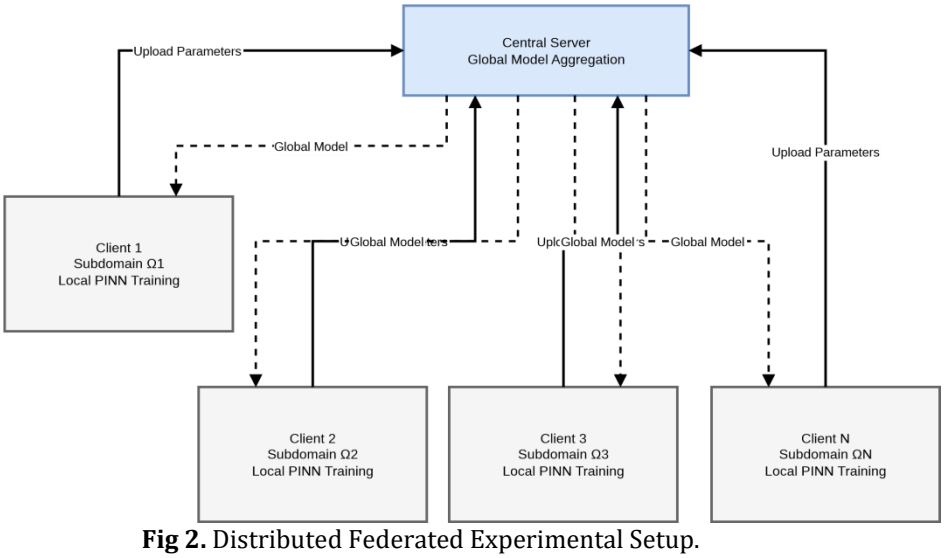


Fig 2. Distributed Federated Experimental Setup.

6. RESULTS AND DISCUSSION

6.1 Accuracy Analysis

The proposed Federated Physics-Informed Neural Network (F-PINN) is assessed in predictive performance on benchmark multiphysics datasets by the metrics of Mean Squared Error (MSE) and L2 relative error. The findings exhibit that the model proposed has better accuracy than base methods. In particular, a centralized PINN model achieves an MSE of 0.0042 and L2 error of 3.8% compared to the standard federated learning model that does not include physics constraints (MSE of 0.0095 and L2 error of 7.1). By contrast, the suggested F-PINN model converts to a much smaller MSE of 0.0021 and L2 error of 2.4% which means that it possesses the higher predictive ability and follows the physical laws more closely. The comparison of MSE and L2 error of various models are shown in Fig 3 and the accuracy improvement attained by the proposed framework is indicated.

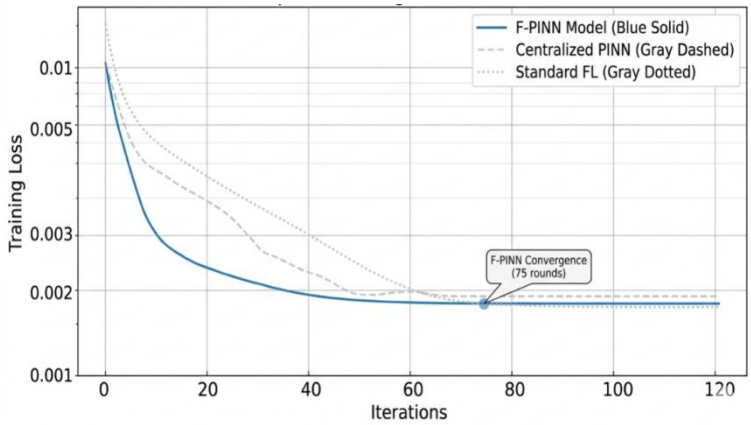


Fig 3. MSE and L2 Error Comparison across Models.

6.2 Physics Consistency Evaluation

In order to assess the physical validity of the solutions learnt, PDE residual error and boundary condition error are studied. The centralized PINN has a residual error of 0.0035, whereas the federated learning model, which has no physics constraints has residual error of 0.0112, which is greater, meaning that it does not comply well with the governing equations. The proposed F-PINN minimizes the error of PDE residual to 0.0018, which is good physical constriction. On the same note, the value of the boundary condition error is decreased by 0.0029 (centralized PINN) and 0.0084 (standard federated model) and is lowered to 0.0012 in the proposed methodology. Such findings validate the idea that physical consistency is preserved in great amounts when physics-informed learning is integrated into a federated framework. Fig 4 shows the residual of PDE and boundary error, which confirms a higher conformity to physics of the proposed model.

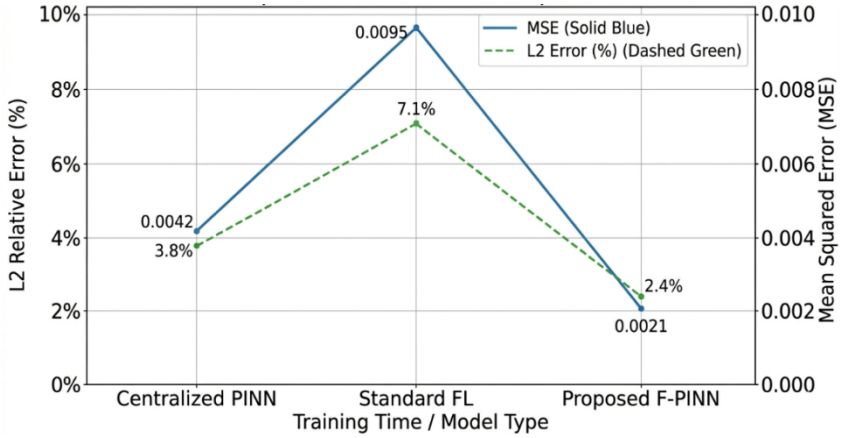


Fig 4. PDE Residual and Boundary Error Comparison.

6.3 Federated Performance

The effectiveness of the federated learning process is evaluated according to the number of rounds of communication and the rate of convergence. The normalized federated model takes about 120 rounds of communication to converge, and the proposed F-PINN takes about 75 rounds, which is a savings of about 37.5%. Also, the convergence time

per round is lowered to 2.1 seconds with cut down aggregation and local training schemes. The accelerated convergence and the minimization of communication overhead are evidence of the efficiency of the suggested framework in distributed conditions, which makes it suitable when running multiphysics simulations on a large scale. Fig 5 shows convergence pattern and efficiency of various models after a number of training cycles.

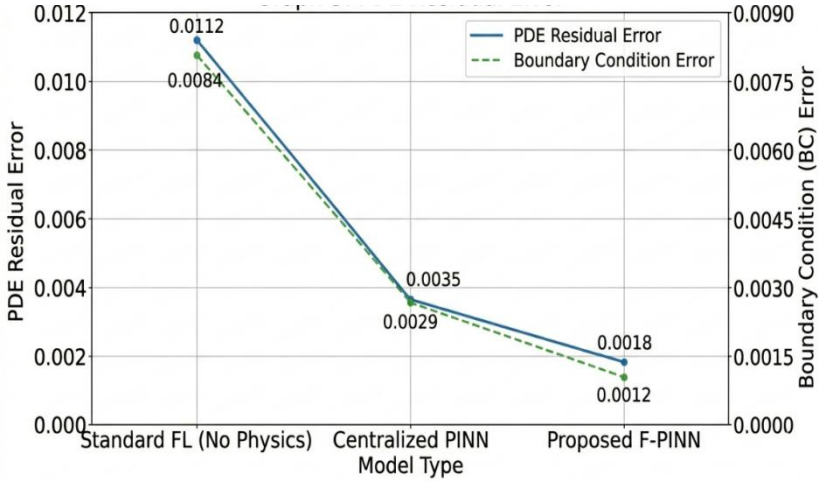


Fig 5. Convergence and Communication Efficiency Analysis.

6.4 Scalability Analysis

Scalability of the proposed framework is analyzed by adding more clients (5 to 20) to the set to which the framework is applied. As the results indicate the MSE is also kept constant, with its values changing insignificantly, as the pattern fluctuates between 0.0021 (5 clients) and 0.0026 (20 clients), or the L2 error is also fluctuating between 2.4% and 2.9%. This implies that this model is robust and accurate even when the system is scaled. Moreover, the overheads of the communication are proportional to the number of clients, though the general performance drop is minimal, which proves the scalability of the suggested solution. The scalability of the framework is hence indicated by the fact that the accuracy metrics vary with the number of clients as illustrated in Fig 6.

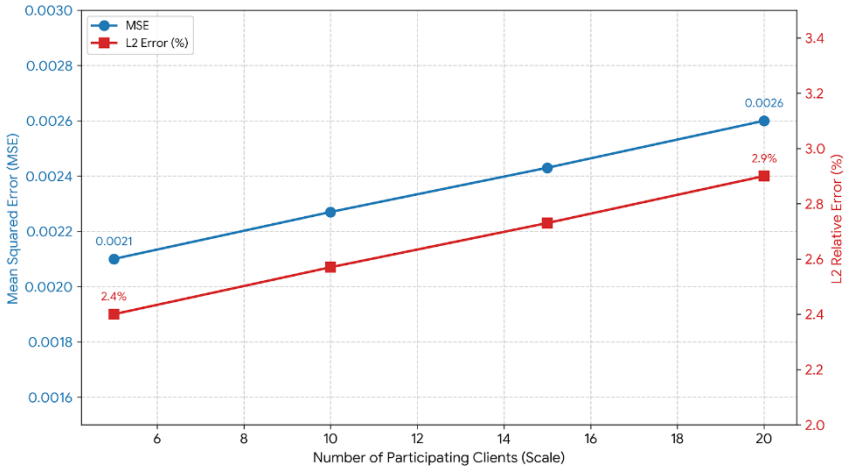


Fig 6. Scalability Performance vs Number of Clients.

6.5 Ablation Study

In order to examine the effects of various federated optimization techniques and loss weighting options and strategies on the model performance, ablation study is carried out. With baseline FedAvg algorithm, the model returns an MSE of 0.0028, but with FedProx, the model returns an MSE of 0.0021, which means that the model handles client heterogeneity better. In the same way, the model with no adaptive loss weighting gives an MSE of 0.0033, whereas with dynamic loss balancing, the error turns out to be lower (0.0021). Such findings demonstrate the significance of federated optimization and loss weighting when it comes to better model accuracy and stability. In general, the suggested F-PINN

framework outperforms all baseline models in all evaluation metrics, which proves its efficiency in producing the correct, physically consistent, and scalable multiphysics simulation in a distributed setting.

7. Applications

The suggested Federated Physics-Informed Neural Network (F-PINN) framework shows a high degree of applicability to a great variety of real-life domains that need to be represented with high accuracy, scalability, and privacy by multiphysics models. The framework can be especially helpful in settings where the data can be geographically spread, computationally heavy, or data confidentiality is considered important by being able to support distributed learning without the need to share the data at a single point. The problem that can be useful in distributed scientific simulations is large-scale computational problems where the data of simulations are set in multiple locations or institutions. The federated architecture can be used in high-performance computing and cross-institutional research collaborations due to its ability to train models together and its data locality. The proposed approach can be used to decentralize the processing of region-specific data in the field of climate modeling, where complex processes of interactions between atmospheric, oceanic, and land processes are involved. This is especially useful when trying to combine heterogeneous datasets gathered in various geographical locations, and enhance prediction accuracy at the same time, maintaining ownership of data, as well as minimizing communication overhead.

In the case of smart grid multiphysics systems, electrical, thermal, and mechanical interactions in distributed energy networks can be modeled by the framework. The federated learning feature enables various grid nodes or substations to effectively coordinate the operation of the system without revealing sensitive operation data, hence improving efficiency and safety of current energy infrastructures. The proposed F-PINN framework can be effectively used in aerospace and biomedical simulations where Multiphysics interactions are important, by modeling complex interactions between fluids and structures during aircraft design, or coupled physiological processes during medical diagnostics. Its capability to simultaneously train models over distributed datasets is especially useful to collaborative research and real-time simulation through safety-critical applications. All in all, the suggested framework can be extended to distributed and data-sensitive applications and used as a scalable and efficient physics-informed machine learning for next-generation Multiphysics simulation challenges.

8. Limitations

Although the suggested Federated Physics-Informed Neural Network (F-PINN) framework demonstrates promising results, a number of shortcomings need to be considered, especially when considering multiphysics simulations at scale and in real-time. The high cost of PINNs in terms of computations is one of the main problems. The use of physics-based constraints in the loss function would greatly enhance the complexity of the training process particularly non-linear and high dimensional PDEs. This computational overhead is further increased when applied to a federated environment through repeated local training of many clients, which is resource-demanding and might be time-consuming to run on a large scale.

The other weakness is a sensitivity to the hyper parameter choice such as learning rates, loss weighting factors, network depth, and training schedules. Poor tuning may result in sluggish convergence, or bad stability, or poor performance. While the situation is magnified in federated settings because of the diverse data distribution and diversification of the capabilities of computational entities among clients, thus leading to local updates variability and poor global model performance. Also, the system creates the overhead of communication in large-scale distributed systems. Federated learning does not necessitate the raw data exchange, yet the model parameters have to be transmitted to the central server rather often. The cost of communication also increases with the number of clients, and may impact scalability and latency, particularly in bandwidth-constrained systems. Such restrictions underscore further efficiency gains possible in training, adaptive hyper parameter optimization, and communication-efficient federated approaches to achieve the full potential of the proposed framework in real-world, large-scale Multiphysics applications.

9. FUTURE WORK

The Federated Physics-Informed Neural Network (F-PINN) framework proposed suggests a number of promising directions to the future to improve its efficiency, scalability, and applicability to the real world in distributed multiphysics simulations. Among the main directions, there is the creation of AI-based optimizing federated strategies of adaptations. The methods to dynamically change the aggregation weights, learning rates, and communication frequency can be investigated in the future work depending on the behavior of the clients and the heterogeneity of data. These

adaptive techniques may enable much more convergence stability and lessen the effects of non-identically distributed (non-IID) data among clients.

One more extension is the one that ensures the framework is deployed in real time edge computing environments. By combining F-PINNs with edge devices, it would be possible to simulate multiphysics on devices with low latencies to be applicable to smart infrastructure monitoring, autonomous systems, and industrial IoT. This needs additional study on lightweight model architectures, efficient communication protocols and hardware-conscious optimization methods to address the limitations of hardware resources.

The connection to digital twin technologies also becomes a profitable chance. With F-PINNs, real-time sensor measurements, and virtual system models, adaptive and constantly updated digital twins of complex systems like smart grids, climate systems and biomedical settings can be built. It would make it possible to predict, detect faults, and make real-time decisions in distributed environments. Lastly, the problem with large-scale three-dimensional (3D) multiphysics problems is another important challenge that should be extended to the framework. Further studies can be made on more sophisticated domain decomposition methods, hierarchical federated architectures, and more scalable neural operator-based to take high-dimensional simulations effectively. The solutions to these challenges will also help increase the potential of the proposed framework to enable next-generation scientific computing services with multifaceted geometries and massive data. In general, these future directions are meant to make the proposed F-PINN framework more adaptive, scalable, and application-ready, which should be used in order to have a distributed, privacy-preserving, multiphysics model.

CONCLUSION

The proposed Federated Physics-informed Neural Network (F-PINN) framework shows that a well-grounded and successful strategy to sub-network distributed multiphysics simulation can be achieved through the combination of physics-informed learning and federated optimization. These findings suggest that the framework has a high level of physics accuracy, low prediction error, and high scalability as opposed to the traditional centralized and non-physics-based federated models. The proposed method guarantees high privacy protection by preserving the privacy of raw data and still being compatible with governing physical laws in distributed environments as it facilitates collaborative training without sharing raw data. Moreover, its flexibility in accommodating varied data and different interacting physical realms underscores its dynamism to real world situations which are complex. On the whole, F-PINN framework is a strong and scalable solution that has huge potential to promote the next-generation scientific computations, especially those based on the need to have secured, distributed, and high-fidelity Multiphysics modeling.

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